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# A study of muons underground and their energy spectrum at sea level

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**Abstract.** An expression $I(E, h)$ relating the integral intensity of muons and depth underground is presented. Particular attention is given to quantifying the various sources of uncertainty relevant to the accuracy of this expression. Measurements of sea-level intensities made with magnetic spectrographs are compared with the predictions of underground results in the energy range 5–1000 GeV.

## 1. Introduction

Although the relevance of underground intensity measurements at shallow and moderate depths (sea level to 1000 hg cm$^{-2}$) to the sea-level muon spectrum, as measured by magnetic spectrographs, has long been appreciated, there has been no recent critical comparison. It is customary (see, for example, Kobayakawa 1967 and Ng and Wolfendale 1974) to compare spectrograph results with the predictions of underground measurements only in the high energy region $10^2$–$10^4$ GeV. Admittedly, this is an energy region of great interest, but unfortunately comparison and hence conclusions are masked by large errors in both spectrograph and the underground measurements relevant to these energies. The absolute integral sea-level muon intensity, as given by Rossi (1948), was generally accepted until recently when Allkofer *et al* (1971) obtained a result some 26% higher. Similar measurements by Ayre *et al* (1973) and Crookes and Rastin (1972), although higher than the Rossi value, are about 10% lower than the result of Allkofer *et al.* Such mutual disagreement is interpreted as a measure of the uncertainty in absolute sea-level intensities at the present time.

Analysis will show that the intensities predicted from underground experiments are particularly reliable in the range $5 < E < 100$ GeV and therefore it is pertinent to compare sea-level intensities, derived from shallow–moderate depth measurements, with the corresponding spectrograph results.

## 2. Depth–intensity measurements

The scatter exhibited by the data collected together in comprehensive surveys such as those of Menon and Ramana Murthy (1967) and Barton and Stockel (1968) would appear to preclude the extraction of a reliable depth–intensity curve. However, these surveys are dominated at shallow depths by results of experiments carried out over twenty years ago using apparatus which would nowadays be regarded as elementary.

An underground survey based on more recent measurements was previously reported by Wright (1973a, to be referred to as I). For this purpose only vertical, absolute results were accepted but converted to standard rock where necessary. It was also shown in I that underwater intensity measurements of Higashi *et al* (1966) and Davitaev *et al* (1969) are compatible with underground results, provided that the correct conversion to standard rock is used. Depth–intensity curves for rock and water are shown in figures 1(*a*) and (*b*). The best fit curve indicated in both figures is the expression of Miyake (1963)

$$I = A_0\left(\frac{h^{-\alpha}\exp(-\beta h)}{h+h_1}\right)\,\mathrm{s^{-1}\,cm^{-2}\,sr^{-1}}. \qquad (1)$$

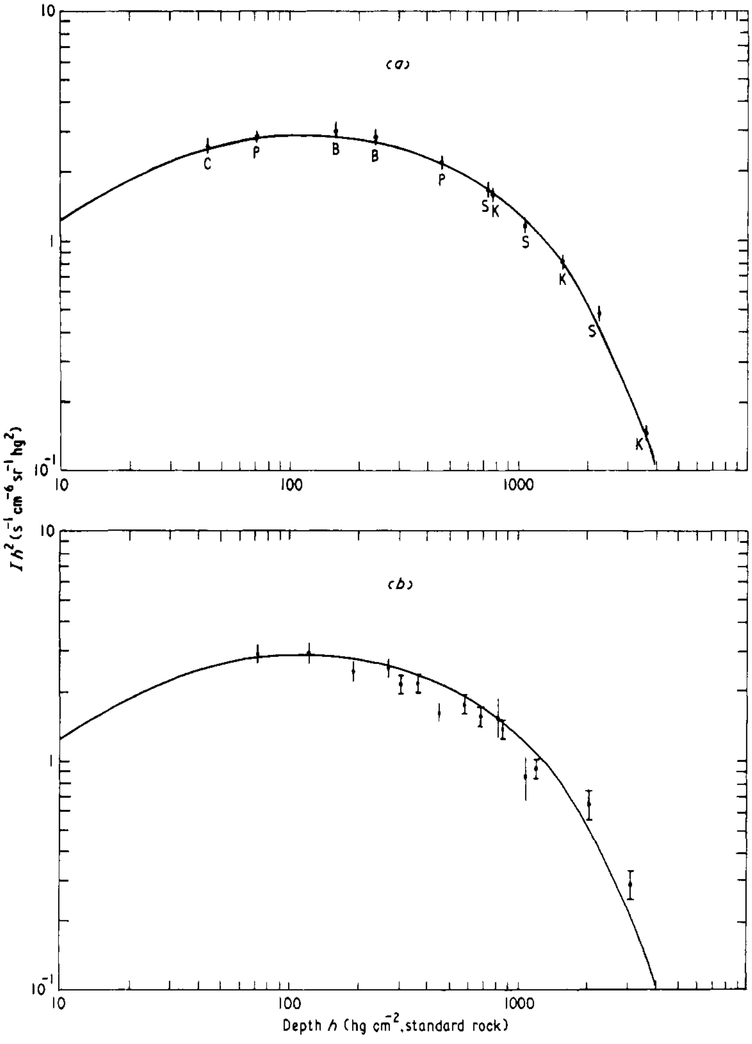


**Figure 1.** Intensity–depth curves for (*a*) rock and (*b*) water. All depths are measured from the top of the atmosphere. Experimental points: C, from Crookes and Rastin (1971); P, from Wright (1973a); S, from Stockel (1969); B, from Bergamasco and Picchi (1971): K, from Krishnaswamy *et al* (1969); ɸ, from Higashi *et al* (1966); Ɨ, from Davitaev *et al* (1969).

The parameters in (1) were determined by a least squares technique applied to the underground intensities only and gave $A_0 = 8{\cdot}0, \alpha = 1{\cdot}13, h_1 = 35{\cdot}0$ and $\beta = 8{\cdot}25 \times 10^{-4}$. Expression (1) with these same parameters is compared with the underwater results in figure 1(*b*); the underwater intensities have been converted to equivalent depths of standard rock using the method reported in I. From figure 1(*b*) it is clear that experimental points between 100 and 1000 hg cm$^{-2}$ tend to be lower than the underground measurements, but this is not considered significant. A careful consideration of the errors and uncertainties which enter into any depth–intensity relationship is essential if (1) is to have any meaning.

(i) Apparatus aperture and geometry. This has been fully discussed in I where it was shown that absolute intensity measurements may be made to an accuracy of about 2 %.

(ii) Effects of incident showers. These have been shown in I to be relatively unimportant at depths less than 1000 hg cm$^{-2}$ but are considered seriously to limit the accuracy of measurements at great depths to no better than 5 %.

(iii) Depth in hg cm$^{-2}$. The depth of an apparatus in feet or metres below the surface is often known very accurately, particularly where the location is a mine. In terms of hg cm$^{-2}$, the accuracy of the depth is clearly as good as $\delta\rho/\rho$, the uncertainty in the rock density. For example, even though detailed data in the form of gravimetric and geological surveys is available for the sites used by Stockel (1969) and Krishnaswamy *et al* (1969) (Kolar Gold Fields), the uncertainty in density is quoted as 1 % and 2 % respectively. Over a restricted range, $I$ and $h$ may be related through a power law $I = Ah^{-m}$ and hence $\delta I/I \simeq m\delta h/h$ where $m$ is the log slope of the $I$–$h$ curve. Any error in $h$ due to $\delta\rho/\rho$ may thus be absorbed as an equivalent error $\delta I/I$, as shown in table 1.

**Table 1.** Errors in intensity resulting from uncertainties in rock density. The figures in brackets are the approximate range energies.

| $h$(hg cm$^{-2}$) | $m$ | $\delta\rho/\rho$ (%) | $\delta I/I$(%) |
|---|---|---|---|
| <500 ($10^2$ GeV) | 2 | 2 | 4 |
| 2500 ($10^3$ GeV) | 4 | 2 | 8 |
| 7000 ($10^4$ GeV) | 8 | 2 | 16 |

(iv) Conversion to standard rock. Depths at sites where $Z$ and $A$ are different from 11 and 22 respectively need conversion to standard rock. It is clear from figure (4) of I that the composition of the rock is not critical when depths of less than 1000 hg cm$^{-2}$ are considered. At greater depths, where radiation losses dominate, an exact knowledge of $Z$ and $A$ is important.

## 3. Energy loss processes and the range–energy relationship

The range of a muon in standard rock with initial energy $E$ is given by computing the following integral for $Z = 11$ and $A = 22$:

$$R(E) = \int_0^E \frac{\mathrm{d}E}{a(E)+b(E)E}\ \mathrm{g\,cm^{-2}} \qquad (2)$$

where $a(E)$ represents ionization losses, $b(E) = b_{\mathrm{P}}(E)+b_{\mathrm{B}}(E)+b_{\mathrm{N}}(E)$ accounts for pair production, bremsstrahlung and nuclear interactions. Many computations of (2) have been made, recently by Kotov and Logunov (1969), Kobayakawa (1967, 1973), Barton and Stockel (1969) and Bergamasco and Picchi (1971), but it is considered necessary to repeat the calculation using the latest cross sections for $b(E)$ and the complete formula for $a(E)$.

### 3.1. *Ionization loss*

$$a(E) = \frac{A}{\beta^2}\left(B + 0{\cdot}69 + 2\ln\frac{P}{\mu} + \ln E'_m - 2\beta^2 - \delta\right)\ \mathrm{MeV\,g^{-1}\,cm^2} \qquad (3)$$

(from Sternheimer 1956) which has been shown experimentally to be correct to better than 1% (Crispin and Fowler 1970). The frequently used approximate relation, derived from (3)

$$a(E) = 1{\cdot}888 + 0{\cdot}0768\ln E'_m/\mu\ \mathrm{MeV\,g^{-1}\,cm^2} \qquad (4)$$

gives acceptable agreement with (3), of the order of 1%, only when $E > 100$ GeV.

### 3.2. *Bremsstrahlung*

The term $b_B(E)$ was evaluated by numerical integration of the differential cross section of Petrukhin and Shestakov (1968). Following Kobayakawa (1967), it is convenient to express the energy loss in the following empirical form:

$$b_B(E) = \begin{cases} (0{\cdot}184\ln E/\mu - 0{\cdot}10)\times 10^{-6}\ \mathrm{g^{-1}\,cm^2} & E \leqslant 10^2\ \mathrm{GeV} \\ 2{\cdot}29\left(\dfrac{\ln E/\mu - 4{\cdot}00}{\ln E/\mu - 1{\cdot}26}\right)\times 10^{-6}\ \mathrm{g^{-1}\,cm^2} & 10^2 < E < 10^4\ \mathrm{GeV}. \end{cases} \qquad (5)$$

### 3.3. *Direct pair production*

The energy loss computed from the cross section of Kelner and Kotov (1968) given by Wright (1973b) is

$$b_P(E) = \begin{cases} (0{\cdot}37\ln E/\mu - 0{\cdot}95)\times 10^{-6}\ \mathrm{g^{-1}\,cm^2} & E < 10^2\ \mathrm{GeV} \\ 2{\cdot}75\left(\dfrac{\ln E/\mu - 5{\cdot}43}{\ln E/\mu - 4{\cdot}34}\right)\times 10^{-6}\ \mathrm{g^{-1}\,cm^2} & 10^2 < E < 5\times 10^4\ \mathrm{GeV}. \end{cases} \qquad (6)$$

### 3.4. *Nuclear losses*

The magnitude of $b_N(E)$ was previously uncertain to about 50%, but recent accelerator experiments of Caldwell *et al* (1970) have now established the photonuclear cross section as $120 \pm 10$ μb. Kobayakawa (1973) has recalculated $b_N(E)$ on the basis of the higher $\sigma_\gamma$ values; the following empirical relationship may be formulated from his work:

$$b_N(E) = (0{\cdot}423 + 0{\cdot}0081\ln E/\mu)\times 10^{-6}\ \mathrm{g^{-1}\,cm^2}. \qquad (7)$$

This expression is in good agreement with the experimental result of

$$0{\cdot}57 \pm 0{\cdot}11\times 10^{-6}\ \mathrm{g^{-1}\,cm^2}$$

determined by Bezrukov *et al* (1972).

The contributions from orbital electrons are included in both (5) and (6) but, according to Rozental (1968), uncertainties arising from the physical assumptions made in the derivation of the cross sections limit the accuracy to about 5%. This is considerably more than the difference between the emprical formulae (5) and (6) and the actual computed values of $b_B(E)$ and $b_P(E)$ from which they were derived. It is clear from the work of Cassiday (1971) that there is still doubt about the physics of the nuclear process (7), especially the dependence of the cross section on $A$ and $q^2$, so that (7) probably has associated with it as much as $\pm 20\%$ uncertainty.

Substituting (3), (5), (6) and (7) into (2) and integrating numerically gives mean muon ranges for various initial energies. Such computations lead to the following empirical range–energy relationship for $E > 10^4$ MeV:

$$R(E) = \left[\frac{1}{b}\ln\left(1+\frac{b}{a}E\right)\right]\left[0.96\left(\frac{\ln E - 7.894}{\ln E - 8.074}\right)\right] \text{ g cm}^{-2} \qquad (8)$$

where $a = 2.2 \text{ MeV g}^{-1}\text{ cm}^2$, $b = 4.4 \times 10^{-6}\text{ g}^{-1}\text{ cm}^2$ and $E$ is measured in MeV. The first term in square brackets is the well known result obtained by integrating (2) with $a(E)$ and $b(E)$ taken as constants; the second expression is a correction factor. In practice the validity of (8) is restricted by uncertainties in the quantities $Z/A$ and $Z^2/A$ which enter into the expressions for ionization and radiation losses respectively. The survey of Mando and Ronchi (1952) indicates that $Z/A$ does not vary by more than 1% for most rock commonly encountered, but clearly $Z^2/A$ is sensitive to an exact knowledge of $Z$. A more serious source of error lies in an uncertainty of about 10% in $b(E)$ because of the limited precision of the radiation cross sections. The manner in which this limits the accuracy of the range–energy relationship is shown in figure 2.

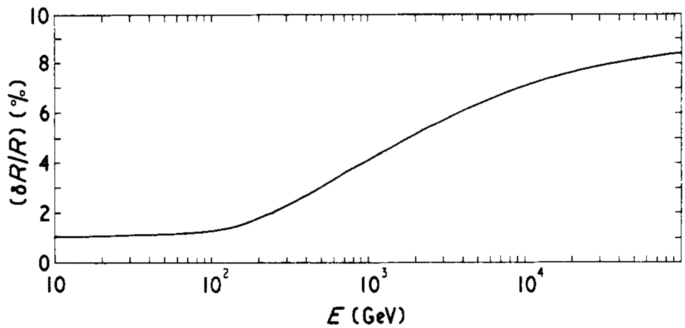


**Figure 2.** The uncertainty in muon range $R$ due to uncertainties in cross sections of 5% each in $b_B(E)$ and $b_P(E)$, 20% in $b_N(E)$ and 1% in $a(E)$.

## 4. The muon energy spectrum at depth $h$ below the top of the atmosphere

An expression for the integral muon spectrum $I(E, h)$ may be derived by combining $R(E)$ and the depth–intensity relationship. The vertical intensity at depth $h$ is

$$I(E, h) = \frac{A_0(h+R(E))^{-\alpha}\exp[-\beta(h+R(E))]}{(h+R(E))+h_1}. \qquad (9)$$

This equation assumes a fixed range for muons of a given energy, which is true only for energies less than about 600 GeV—the critical energy in rock. Intensities at energies greater than 600 GeV have therefore been corrected for range fluctuations, using the results of Kobayakawa (1973), as indicated in figure 3.

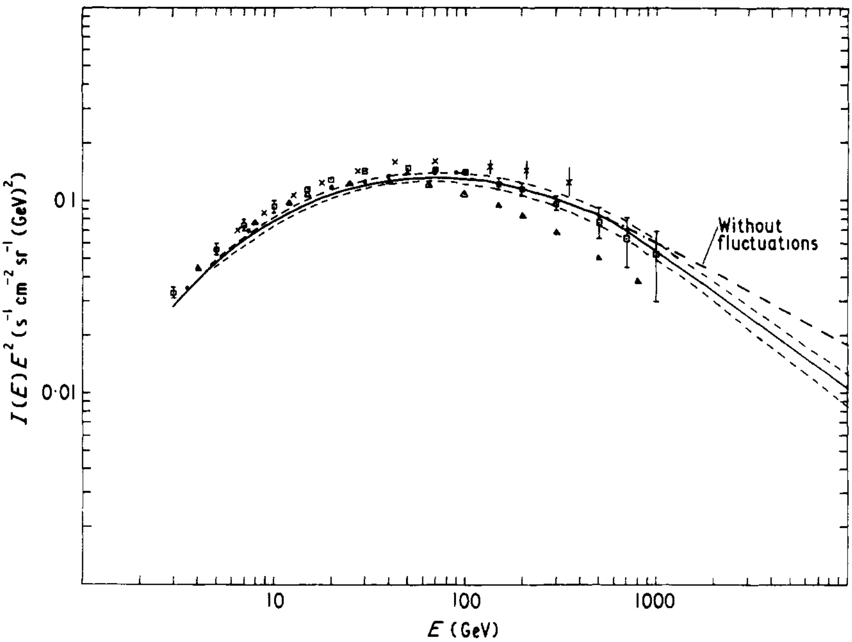


**Figure 3.** Comparison of integral sea-level energy spectrum, deduced from underground measurements (full curve with broken curve one $\sigma$ limits), with direct magnetic spectrograph measurements: ⊡, Allkofer *et al* (1971); ▲, Appleton *et al* (1971) 'best-fit' curve; ×, Nandi and Sinha (1972); ● Ayre *et al* (1973).

The vertical sea-level spectrum predicted by (9) is of special interest and is obtained by taking $h = 10{\cdot}3$ hg cm$^{-2}$. The spectrum so derived is shown in figure 3, together with more recent spectrograph results. The Nottingham results were taken from table 10 of Appleton *et al* (1971) and have been renormalized to an intensity of $9{\cdot}13 \times 10^{-3}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ at $E > 0{\cdot}35$ GeV, in accordance with the later absolute measurement of Crookes and Rastin (1972). The lines of one standard deviation indicated follow from table 2 where estimates of the various contributions are discussed in the previous sections. Those error estimates listed under 'apparatus' are based on a telescope of semi-cubical geometry, as employed by the author, and do not necessarily apply to other configurations.

## 5. Conclusions

It has been demonstrated that underground measurements at moderate depths can provide an accurate sea-level muon spectrum in the range 5–100 GeV. Measurements at greater depths allow the energy range to be extended but with decreasing accuracy. Apart from the results of Ayre *et al* (1973), all other spectrograph results shown indicate

**Table 2.** Error contributions to the sea-level spectrum.

| $E$ (GeV) | Apparatus (%) | $\delta h/h$ (%) | $\delta R/R$ (%) | Total (%) |
|---|---|---|---|---|
| 5 | 2 | 4 | 1·5 | 4·5 |
| 10 | 2 | 4 | 1·5 | 4·5 |
| 50 | 3 | 4 | 1·5 | 5 |
| 100 | 3 | 5 | 1·6 | 6 |
| 500 | 4 | 7 | 3·5 | 7 |
| 1000 | 5 | 8 | 4·5 | 10 |
| 5000 | 6 | 12 | 6·5 | 15 |
| 10000 | 8 | 16 | 7·0 | 19 |

significantly different intensities than predicted by the underground measurements. The results of Allkofer *et al* (1971) and Nandi and Sinha (1972) are high with respect to the underground measurements, while those of Appleton *et al* (1972) are low in the region $E > 50$ GeV. However, it is evident in figures 1–3 of Appleton *et al* that a similar discrepancy is present between their experimental points and their best fit curve in the energy region $E > 100$ GeV. Agreement with all results demands extending the confidence limits to $\pm 20\%$ in the region $5 < E < 70$ GeV. Admittedly the error contributions in table 2 are only estimates, but it is difficult to accept that they have been underestimated by a factor of four. It is significant that the spectrograph measurements follow the same shape curve as in figure 3, needing only renormalization of between 5 and 25% in order to show good agreement with underground predictions.

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